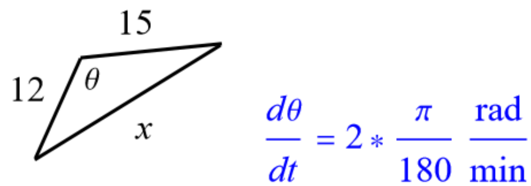
Exercise 41

Two sides of a triangle have lengths 12 m and 15 m. The angle between them is increasing at a rate of 2°/min. How fast is the length of the third side increasing when the angle between the sides of fixed length is 60°?

Solution

Draw a schematic of the triangle at a certain time.



The aim is to find dx/dt when $\theta = 60° = \pi/3$. The relationship between x and θ is given by the law of cosines.

$$x^2 = 12^2 + 15^2 - 2(12)(15)\cos\theta$$

Take the derivative of both sides with respect to time by using the chain rule.

$$\frac{d}{dt}(x^2) = \frac{d}{dt}[12^2 + 15^2 - 2(12)(15)\cos\theta]$$

$$2x \cdot \frac{dx}{dt} = -2(12)(15)(-\sin\theta) \cdot \frac{d\theta}{dt}$$

Solve for dx/dt.

$$\begin{aligned}\frac{dx}{dt} &= \frac{180\sin\theta}{x}\frac{d\theta}{dt} \\ &= \frac{180\sin\theta}{\sqrt{12^2 + 15^2 - 2(12)(15)\cos\theta}}\left(2 \times \frac{\pi}{180}\right) \\ &= \frac{2\pi\sin\theta}{\sqrt{369 - 360\cos\theta}}\end{aligned}$$

Therefore, when the angle between the sides of fixed length is 60°, the rate that the length of the third side changes with respect to time is

$$\left.\frac{dx}{dt}\right|_{\theta=\pi/3} = \frac{2\pi\sin\frac{\pi}{3}}{\sqrt{369 - 360\cos\frac{\pi}{3}}} = \frac{\pi}{3\sqrt{7}}\ \frac{\text{meters}}{\text{minute}} \approx 0.395803\ \frac{\text{meters}}{\text{minute}}$$